MA1407.000500 Mathematical Algorithms II Assignment 4

Fall 2025

Out: 25th November 2025

Due: 9th December 2025 at 11:59pm

Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will **not** be accepted except in extenuating circumstances. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

Question:	1	2	Total
Points:	50	50	100

1. (50 points) Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. For each edge $e \in E$, let $w_e \in \mathbb{R}_+$ be the (nonnegative) weight of e. Let OPT denote the maximum weight of a cut in G, which can be computed by the binary quadratic optimization formulation

$$\begin{aligned} \text{maximize} \quad & \sum_{e=\{u,v\}\in E} \frac{w_e(1 - x_u x_v)}{2} \\ \text{subject to} \quad & x \in \{-1, 1\}^{|V|}. \end{aligned}$$

Suppose that X^* be an optimal solution to its SDP relaxation, given by

$$\begin{aligned} \text{maximize} \quad & \sum_{e=\{u,v\}\in E} \frac{w_e(1 - X_{uv})}{2} \\ \text{subject to} \quad & X_{vv} = 1 \quad \forall v \in V, \\ & X \succeq 0. \end{aligned}$$

Then take the Cholesky decomposition of X^* given by $X^* = Y^\top Y$, and let y_v for $v \in V$ denote the columns of Y. Next, we take a vector g from the unit sphere $\{x \in \mathbb{R}^n : \|x\|_2 = 1\}$ uniformly at random. Take $S = \{v \in V : g^\top y_v \geq 0\}$. Prove that

$$\mathbb{E}\left[|\delta(S, V \setminus S)|\right] \geq 0.878 \times \text{OPT}.$$

2. (50 points) Let $f_1, \ldots, f_T$ be an arbitrary sequence of convex loss functions that are L-Lipschitz in a norm $\|\cdot\|$. Assume that the Bregman divergence D_ψ satisfies

$$D_\psi(x, y) \geq \frac{1}{2}\|x - y\|^2$$

for any $x, y \in C$. Moreover, $R^2 = \sup_{x,y\in C} D_\psi(x, y)$. Prove that online mirror descent with step sizes $\eta_t = R/(L\sqrt{t})$ guarantees that

$$\sum_{t=1}^{T} f_t(x_t) - \min_{x\in C} \sum_{t=1}^{T} f_t(x) = O\left(LR\sqrt{T}\right).$$