MA1407.000500 Mathematical Algorithms II Assignment 3

Fall 2025

Out: 30th October 2025

Due: 12th November 2025 at 11:59pm

Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will **not** be accepted except in extenuating circumstances. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

Question:	1	2	Total
Points:	50	50	100

1. (50 points) Let $\phi : X \times Y \to \mathbb{R}$ be a L-Lipschitz continuous convex-concave function. Assume that $\|x_1 - x_2\|_2 \leq R$ for any $x_1, x_2 \in X$ and $\|y_1 - y_2\|_2 \leq R$ for any $y_1, y_2 \in Y$. Prove that gradient descent ascent with step size $\eta = R/L\sqrt{T}$ guarantees that for any $(x, y) \in X \times Y$,

$$\phi\left(\frac{1}{T}\sum_{t=1}^{t} x_t, y\right) - \phi\left(x, \frac{1}{T}\sum_{t=1}^{t} y_t\right) \leq \frac{2LR}{\sqrt{T}}.$$

2. (50 points) Recall that optimistic gradient descent ascent proceeds with the following update rule.

$$\begin{aligned}
x_{t+\frac{1}{2}} &= x_t - \eta \nabla_x \phi\left(x_{t-\frac{1}{2}}, y_{t-\frac{1}{2}}\right), \\
y_{t+\frac{1}{2}} &= y_t + \eta \nabla_y \phi\left(x_{t-\frac{1}{2}}, y_{t-\frac{1}{2}}\right), \\
x_{t+1} &= x_t - \eta \nabla_x \phi\left(x_{t+\frac{1}{2}}, y_{t+\frac{1}{2}}\right), \\
y_{t+1} &= y_t + \eta \nabla_y \phi\left(x_{t+\frac{1}{2}}, y_{t+\frac{1}{2}}\right).
\end{aligned}$$

Prove that the update rule is equivalent to setting $t' = t + \frac{1}{2}$ and performing the following update:

$$\begin{aligned}
x_{t'+1} &= x_{t'} - 2\eta \nabla_x \phi(x_{t'}, y_{t'}) + \eta \nabla_x \phi(x_{t'-1}, y_{t'-1}), \\
y_{t'+1} &= y_{t'} + 2\eta \nabla_y \phi(x_{t'}, y_{t'}) - \eta \nabla_y \phi(x_{t'-1}, y_{t'-1}).
\end{aligned}$$