IE 539: Convex Optimization, Fall 2023

MW(월수) 4:00-5:15 pm, E2-2 (산업경영학동) #B105

Instructor: Dabeen Lee, `dabeenl@kaist.ac.kr`, E2-2 #2109.

Lectures: Mondays and Wednesdays 4:00-5:15 pm.

Office hours: Tuesdays 2:00-3:00 pm.

Teaching assistant: To be announced.

Course webpage: `https://dabeenl.github.io/IE539`

Assignments and lecture notes will be uploaded to the webpage as well as KLMS.

Course description The availability of big data has introduced many opportunities to make better decision-making based on a data-driven approach, and many of the relevant decision-making problems can be posed as optimization models that have special properties such as convexity and smoothness. From this course, a graduate-level student will learn fundamental and comprehensive convex optimization knowledge in theory (convex analysis, optimality conditions, duality) and algorithms (gradient descent and variants, Frank-Wolfe, and proximal methods). We will also cover some application areas including statistical estimation, finance (e.g., portfolio optimization), machine learning, and data science.

Key topics Theory and algorithms for convex optimization, Applications.

- Theory: Convex Analysis (sets, functions, operations), Optimality Conditions, Duality.
- Algorithms: First-order Methods (Gradient Descent and Variants, Frank-Wolfe), Second-order Methods (Newton's method, Quasi-Newton method), Proximal and Dual Methods (Proximal Point Algorithm, ADMM, Douglas-Rachford Splitting), and more.
- Applications: Stochastic and Online Optimization, Statistical Estimation, Finance (e.g., portfolio optimization), Machine Learning and Data Science, Economics.

Texts There is no required textbook for this course, but students may find the following list of materials useful to follow the topics covered in this course:

- Convex Optimization, *Boyd and Vandenberghe*, `https://stanford.edu/~boyd/cvxbook/`
- Convex Optimization: Algorithms and Complexity, *Bubeck*, `https://arxiv.org/abs/1405.4980`
- Lectures on Modern Convex Optimization, *Ben-Tal and Nemirovski*, `https://www2.isye.gatech.edu/~nemirovs/LMCOBookSIAM.pdf`

Prerequisites There are no formal prerequisites but you should be comfortable making mathematical arguments, writing proofs, and programming. You should also be comfortable with the background knowledge from previous courses. Such topics include

- linear algebra (vectors, matrices, inner products, eigenvalues, eigenvectors, singular value decomposition),
- basic multivariate calculus (partial derivatives, gradient), and
- basic mathematical analysis (open and closed sets, inf and sup definitions).

Assessment structure There will be 5 assignments (50%), a course project (20%), and a take-home final exam (30%). **Typesetting in Latex is required for all submissions**.

Course outline What follows is a tentative outline of this course.

- Convex sets and functions.
- Operations preserving convexity
- Quadratic functions, positive semidefinite matrices, differentiable convex functions
- Subgradients of convex functions
- Convex optimization problems and projected subgradient descent
- Stochastic and online gradient decent methods
- Problems with functional constraints and Lagrange duality
- Saddle point problems and the primal-dual subgradient algorithm
- Conic programming
- Conic programming duality
- Newton's method and Quasi-Newton methods
- Semidefinite programming relaxations of quadratic programs
- Frank-Wolfe algorithm
- Alternating direction method of multipliers
- Modeling optimization problems under uncertainty
- Algorithms for optimization under uncertainty
- Applications in machine learning and data science

Course project There are two options for the project.

A Review of a research paper related to convex optimization. Choose a research paper that is related to convex optimization and other topics, **published** in a journal or a conference proceeding. Possible topics include, but are not limited to, optimization algorithms, computational methods and software, optimization methods for machine learning, and applications of optimization. Although this course is on continuous optimization, one can select a paper on discrete optimization **if** there is a direct connection to any material covered in the course.

B Formulation and implementation of algorithms or methods for certain optimization problem. Select a topic that is related to an application of convex optimization or any materials discussed throughout the course. In contrast to the first option, one may take a concrete problem setting from a real world application and implement optimization methods for the problem. Hence, this requires running numerical experiments for some specific problem instances.