IE 331 Operations Research: Optimization Assignment 3

Spring 2023

Out: 27th April 2023

Due: 16th May 2023 at 11:59pm

Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will be subject to a penalty. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

Question:	1	2	3	4	Total
Points:	20	10	30	40	100

1. Let $D = (N, A)$ be a network with two distinct nodes s and t. Suppose that $c_{ij} \geq 0$ for $(i, j) \in A$. Consider the following linear program.

$$\begin{aligned} \min \quad & \sum_{(i,j)\in A} c_{ij} z_{ij} \\ \text{s.t.} \quad & y_i - y_j + z_{ij} \geq 0, \quad (i,j) \in A \\ & y_t - y_s = 1 \\ & z_{ij} \geq 0, \quad (i,j) \in A. \end{aligned} \tag{1}$$ lp

   (a) (10 points) Prove that linear program (1) is the dual of the linear programming formulation for the maximum *st*-flow problem over network $D = (N, A)$.

   (b) (10 points) Use part (a) to prove that (1) has an optimal solution that has integer entries only.

2. (10 points) Let $a = (a_1, \ldots, a_d) \in \{0,1\}^d \setminus \{0\}$ be a fixed nonzero binary vector, and $x = (x_1, \ldots, x_d) \in \{0,1\}^d$ be a vector of binary variables. Then formulate the constraint which prevents $x \geq a$ (i.e., we want to ensure that there is at least on j such that $x_j = 0$ and $a_j = 1$.)

3. Let $x, y \in \{0, 1\}$ be two binary variables.

   (a) (10 points) Model implication
   $$x = 0 \quad \Rightarrow y = 1.$$

   (b) (10 points) Model implication
   $$x = 1 \quad \Rightarrow y = 0.$$

   (c) (10 points) Model implication
   $$x = 0 \quad \Rightarrow y = 0.$$

4. (40 points) Use Gurobi to solve the transportation problem with ramp-up costs with the following model parameters.

	Coordinates	Ramp-up costs	Production capacities
Plant 1	(0, 1.5)	11	70
Plant 2	(2.5, 1.2)	100	60
Plant 3	(1.7, 2.3)	9	60
Plant 4	(0.7, 1.8)	7	50

	Coordinates	Demands
Retailer 1	(0,0)	20
Retailer 2	(0,1)	20
Retailer 3	(0,2)	20
Retailer 4	(1,0)	20
Retailer 5	(1,1)	20
Retailer 6	(1,2)	20
Retailer 7	(2,0)	20
Retailer 8	(2,1)	20
Retailer 9	(2,2)	20

   (a) Report the optimal value (Best objective).

   (b) Report the set of plants in operation according to the optimal solution.