

# IE 331 Operations Research: Optimization Assignment 2

Spring 2023

Out: 28th March 2023

**Due: 11st April 2023 at 11:59pm**

## Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will be subject to a penalty. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

|           |    |    |    |    |    |       |
|-----------|----|----|----|----|----|-------|
| Question: | 1  | 2  | 3  | 4  | 5  | Total |
| Points:   | 10 | 20 | 10 | 15 | 45 | 100   |

1. (10 points) Solve the following linear program with the simplex method.

$$\begin{aligned} \max \quad & z = x_1 + 2x_2 \\ \text{s.t.} \quad & -x_1 + 4x_2 \leq 4 \\ & x_1 - x_2 \leq 3 \\ & x_1, x_2 \geq 0. \end{aligned}$$

2. (20 points) Solve the following linear program with the simplex method.

$$\begin{aligned} \min \quad & z = 6x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 1 \\ & 2x_1 - x_2 \geq 1 \\ & 3x_2 \leq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

3. (10 points) Derive the dual linear program of

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & -x_1 + 4x_2 \leq 4 \\ & x_1 - x_2 \leq 3 \\ & x_1, x_2 \geq 0. \end{aligned}$$

4. (15 points) Derive the dual linear program of

$$\begin{aligned} \min \quad & 6x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 1 \\ & 2x_1 - x_2 = 1 \\ & 3x_2 \leq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

5. In this question, we derive the duals of linear programs in general matrix forms.

(a) (10 points)  $\min \{c^\top x : Ax \geq b, x \geq 0\}$ .

(b) (10 points)  $\max \{c^\top x : Ax \leq b\}$ .

(c) (10 points)  $\max \{c^\top x : Ax = b, x \geq 0\}$ .

(d) (15 points)  $\max \{c^\top x + d^\top z : Ax + Bz \leq b, x \geq 0, z \text{ free}\}$ .