# IE 331 Operations Research: Optimization Assignment 1

## Spring2023

# Out: 14th March 2023 Due: 28th March 2022 at 11:59pm

#### Instructions

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will be subject to a penalty. Special consideration should be applied for in this case.
- It is required that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

Question:	1	2	3	4	Total
Points:	10	40	50	0	100

- 1. (10 points) Prove that  $f(x_1, x_2, x_3) = \max\{3x_1 x_2, 2\max\{x_2, 4x_2 3x_1\} x_1\} 4x_1 + x_2$  is linearly representable by representing its epigraph with a finite system of linear inequalities.
- 2. Remember the problem of finding a center point of a cluster from Lecture 3. There is a cluster of points  $V = \{v^1, \ldots, v^n\} \subseteq \mathbb{R}^d$ . Given a point  $x \in \mathbb{R}^d$ , the distance between x and the cluster V is measured by some distance function  $d : \mathbb{R}^d \to \mathbb{R}_+$ . Then we determine the center of a cluster by solving the following optimization problem.

min 
$$d(x)$$

(a) (10 points) The distance function is given by the sum of the  $\ell_1$ -distance between x and individual vector  $v^i$  for  $i \in [n]$ .

$$d(x) = \sum_{i \in [n]} \|x - v^i\|_1$$

Represent the optimization problem as a linear program.

(b) (10 points) The distance function is given by the sum of the  $\ell_{\infty}$ -distance between x and individual vector  $v^i$  for  $i \in [n]$ .

$$d(x) = \sum_{i \in [n]} \left\| x - v^i \right\|_{\infty}.$$

Represent the optimization problem as a linear program.

(c) (10 points) The distance function is given by the maximum  $\ell_1$ -distance between x and individual vector  $v^i$  for  $i \in [n]$ .

$$d(x) = \max_{i \in [n]} \|x - v^i\|_1.$$

Represent the optimization problem as a linear program.

(d) (10 points) The distance function is given by the maximum  $\ell_{\infty}$ -distance between x and individual vector  $v^i$  for  $i \in [n]$ .

$$d(x) = \max_{i \in [n]} \left\| x - v^i \right\|_{\infty}.$$

Represent the optimization problem as a linear program.

3. (50 points) Write your solutions in LaTeX.

Overleaf website: https://www.overleaf.com

```
\documentclass{article}
     1
     2
                   \usepackage{graphicx} % Required for inserting images
     3
    4
                \title{IE331 Assignment 1}
    5
                 \author{Dabeen Lee}
               \date{\today}
    6
    7
    8 \usepackage{amsmath,amssymb}
   9
 10 < \begin{document}
 11
 12 \maketitle
 13
 14 * \begin{enumerate}
 15
                                \item My answer to question 1 is ...
 16
 17
                              t_{epi}(f) = \left[ \left(x,t\right) \right] + h t \leq r r + h t \leq r + h t
  18
  19 *
                               \item \begin{enumerate}
  20
                                            \item My answer to question 2(a) is ...
  21
 22 *
                                           \begin{align*}
  23
                               \min \quad & f(x) 
 24
                               \text{s.t.}\quad & g_i(x)\leq b_i,\quad i\in[m],\\
                              &x\in\mathbb{R}^d.
  25
 26 \end{align*}
 27
                                            \item My answer to question 2(b) is ...
 28
                                                          $$d(x)=\sum_{i\in[n]}\left\|x-v^i\right\|_\infty.$$
 29
 30
                                            \item My answer to question 2(c) is ...
 31
 32
                                                         d(x)=\sum_{i\in[n]}\left(\left|x-v^i\right|\right]
 33
                                            \item My answer to question 2(d) is ...
 34
                              \end{enumerate}
 35 \end{enumerate}
 36
 37 \end{document}
38
```

## IE331 Assignment 1

#### Dabeen Lee

#### March 14, 2023

1. My answer to question 1 is ...

 $epi(f) = \{(x,t) \in \mathbb{R}^d \times \mathbb{R} : \exists y \in \mathbb{R}^p \text{ s.t. } Ax + Dy + ht \le r\}$ 

2. (a) My answer to question 2(a) is ...

min 
$$f(x)$$
  
s.t.  $g_i(x) \le b_i, \quad i \in [m],$   
 $x \in \mathbb{R}^d.$ 

(b) My answer to question 2(b) is ...

$$d(x) = \sum_{i \in [n]} \left\| x - v^i \right\|_{\infty}.$$

(c) My answer to question 2(c) is ...

$$d(x) = \sum_{i \in [n]} \left\| x - v^i \right\|_1.$$

(d) My answer to question 2(d) is ...

4. (0 points) Download and Install Gurobi and Python. We will ask you to solve optimization models with Gurobi in future assignments.

Gurobi website: https://www.gurobi.com

1. Register via your KAIST email address.

			DOCUMENTAT	TION DOWNLOADS & LICENSES	SUPPORT	MY ACCOUNT
Solutions Y	Resources ∽	Partners ∽	Academic Y	Company 🗸	Free	e Trial Q

2. Obtain an academic license (DOWNLOADS & LICENSES  $\rightarrow$  Your Gurobi Licenses  $\rightarrow$  Request  $\rightarrow$  Named-User Academic)

GUROBI OPTIMIZATION	Gequest License     Free License Options for Acade	emic Users		АСАДЕМИС СОММИ	ERCAL	Documentation • Downloads	🕶 Support 🛩 🏭 🔔 Da
	WLS Academic	Named-User Academic	Academic Cloud	Take Gurobi With You	Educational Institution Site	Online Course	
	\$	۲	$\bigcirc$	Ŵ		<b>N</b>	
	Run Gurobi Optimizer where you want, across or regular machines and container environments (Docker, Kubernetes) with floating capability. • Runs on machines and containers	Get a free, unlimited-use Gurobi Optimizer license for a single person, on a single machine.	Get on-demand access to Gurobi on cloud machines. Although the license is free, you'll be responsible for the cost of cloud resource usage.	Through our Take Gurobi With You (TGWY) program, you can continue to get free, unrestricted access to Gurobi after graduation. This means you can bring the speed.	Share Gurobi with multiple users in your academic departments, schools, and classrooms by running Gurobi on your university's local-area network, at no cost to you or your institution.	If you'd like to learn to use Gurobi, but you don't meet our academic program qualification requirements, you can sign up for aur free Gurobi Online Course License.	
	Runs on machines and containers     Requires an internet connection     Requires an internet connection     dening usage     Con be enterned to a large enternet     Con be enterned to a large enternet     Con be enterned to a large enternet     maintain eligibility	To get started, visit our Help Center and request an Academic Gurobi Cloud License.	scope, and problem-solving power of optimization with you into the workplace, using a tool you already know.	To get started, your institution's network administrator will need to visit our Help Center and request an Academic Site License.	It has limited capabilities (max 2000 decision variables and 2000 constraints)-but it is robust enough for new learners.		

- 3. Download Gurobi Optimizer v<br/>10.0.1 (DOWNLOADS & LICENSES  $\rightarrow$  Download Center<br/>  $\rightarrow$  Gurobi Optimizer)
- 4. We will use Gurobi's Python API. Setup Jupyter Notebook. You may have to download the linux version or the MacOS version of Gurobi.
- 5. (Recommended, but not required) Try out some examples provided by Gurobi written in Python. Gurobi examples: https://www.gurobi.com/jupyter\_models/