# IE 331 Operations Research: Optimization Assignment 1

Spring2023

Out: 14th March 2023

**Due: 28th March 2022 at 11:59pm**

**Instructions**

- Submit a PDF document with your solutions through the assignment portal on KLMS by the due date. Please ensure that your name and student ID are on the front page.
- Late assignments will be subject to a penalty. Special consideration should be applied for in this case.
- It is **required** that you typeset your solutions in LaTeX. Handwritten solutions will not be accepted.
- Spend some time ensuring your arguments are **coherent** and your solutions **clearly** communicate your ideas.

| Question: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 10 | 40 | 50 | 0 | 100 |

1. (10 points) Prove that $f(x_1, x_2, x_3) = \max\{3x_1 - x_2,\ 2\max\{x_2,\ 4x_2 - 3x_1\} - x_1\} - 4x_1 + x_2$ is linearly representable by representing its epigraph with a finite system of linear inequalities.

2. Remember the problem of finding a center point of a cluster from Lecture 3. There is a cluster of points $V = \left\{v^1, \ldots, v^n\right\} \subseteq \mathbb{R}^d$. Given a point $x \in \mathbb{R}^d$, the distance between $x$ and the cluster $V$ is measured by some distance function $d : \mathbb{R}^d \to \mathbb{R}_+$. Then we determine the center of a cluster by solving the following optimization problem.

$$\min_x \quad d(x).$$

   (a) (10 points) The distance function is given by the sum of the $\ell_1$-distance between $x$ and individual vector $v^i$ for $i \in [n]$.

$$d(x) = \sum_{i \in [n]} \left\| x - v^i \right\|_1 .$$

   Represent the optimization problem as a linear program.

   (b) (10 points) The distance function is given by the sum of the $\ell_\infty$-distance between $x$ and individual vector $v^i$ for $i \in [n]$.

$$d(x) = \sum_{i \in [n]} \left\| x - v^i \right\|_\infty .$$

   Represent the optimization problem as a linear program.

   (c) (10 points) The distance function is given by the maximum $\ell_1$-distance between $x$ and individual vector $v^i$ for $i \in [n]$.

$$d(x) = \max_{i \in [n]} \left\| x - v^i \right\|_1 .$$

   Represent the optimization problem as a linear program.

   (d) (10 points) The distance function is given by the maximum $\ell_\infty$-distance between $x$ and individual vector $v^i$ for $i \in [n]$.

$$d(x) = \max_{i \in [n]} \left\| x - v^i \right\|_\infty .$$

   Represent the optimization problem as a linear program.

3. (50 points) Write your solutions in LaTeX.
   Overleaf website: https://www.overleaf.com

```
\documentclass{article}
\usepackage{graphicx} % Required for inserting images

\title{IE331 Assignment 1}
\author{Dabeen Lee}
\date{\today}

\usepackage{amsmath,amssymb}

\begin{document}

\maketitle

\begin{enumerate}
    \item My answer to question 1 is ...

    $$\text{epi}(f) = \left\{(x,t)\in\mathbb{R}^d\times\mathbb{R}:\ \exists y\in\mathbb{R}^p\text{ s.t. } Ax + Dy + h t \leq r \right\}$$

    \item \begin{enumerate}
        \item My answer to question 2(a) is ...

        \begin{align*}
    \min\quad & f(x)\\
    \text{s.t.}\quad & g_i(x)\leq b_i,\quad i\in[m],\\
    &x\in\mathbb{R}^d.
\end{align*}
        \item My answer to question 2(b) is ...

            $$d(x)=\sum_{i\in[n]}\left\|x-v^i\right\|_\infty.$$
        \item My answer to question 2(c) is ...

            $$d(x)=\sum_{i\in[n]}\left\|x-v^i\right\|_1.$$
        \item My answer to question 2(d) is ...
    \end{enumerate}
\end{enumerate}

\end{document}
```

IE331 Assignment 1

Dabeen Lee

March 14, 2023

1. My answer to question 1 is ...

$$\text{epi}(f) = \left\{(x,t)\in\mathbb{R}^d\times\mathbb{R}:\ \exists y\in\mathbb{R}^p\text{ s.t. } Ax + Dy + h t \leq r \right\}$$

2. (a) My answer to question 2(a) is ...

$$\begin{aligned} \min\quad & f(x)\\ \text{s.t.}\quad & g_i(x)\leq b_i,\quad i\in[m],\\ &x\in\mathbb{R}^d. \end{aligned}$$

   (b) My answer to question 2(b) is ...

$$d(x)=\sum_{i\in[n]}\left\|x-v^i\right\|_\infty.$$

   (c) My answer to question 2(c) is ...

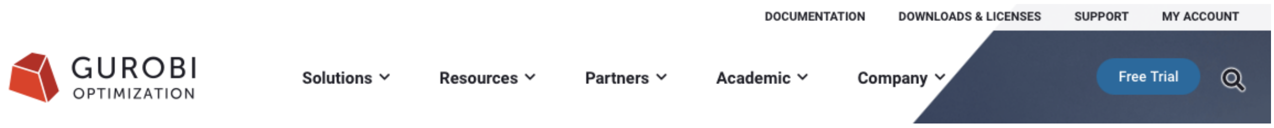
$$d(x)=\sum_{i\in[n]}\left\|x-v^i\right\|_1.$$

   (d) My answer to question 2(d) is ...

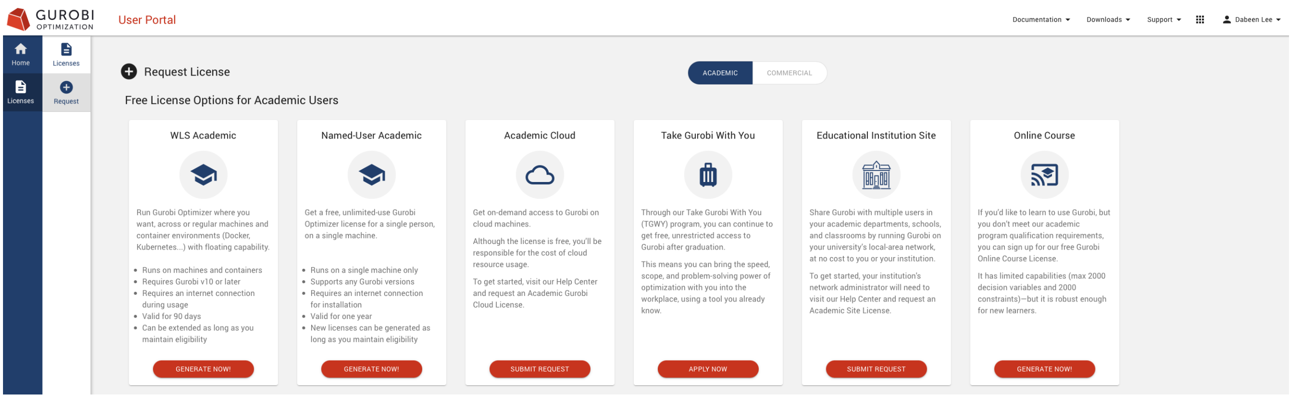
4. (0 points) Download and Install Gurobi and Python. We will ask you to solve optimization models with Gurobi in future assignments.

   Gurobi website: https://www.gurobi.com

   1. Register via your KAIST email address.




   2. Obtain an academic license (DOWNLOADS & LICENSES → Your Gurobi Licenses → Request → Named-User Academic)




   3. Download Gurobi Optimizer v10.0.1 (DOWNLOADS & LICENSES → Download Center → Gurobi Optimizer)
   4. We will use Gurobi's Python API. Setup Jupyter Notebook. You may have to download the linux version or the MacOS version of Gurobi.
   5. (Recommended, but not required) Try out some examples provided by Gurobi written in Python. Gurobi examples: https://www.gurobi.com/jupyter_models/